Preface

This textbook is intended to be an introduction to complex variables for engineering and science undergraduate students. The prerequisites are some knowledge of calculus (up to line integrals and Green's Theorem), though basic familiarity with differential equations would also be useful.

Complex variable theory is an elegant mathematical structure on its own. On the other hand, it provides powerful tools for solving problems elsewhere in mathematics and the physical sciences. In this book, I have emphasized how to *use* complex variable methods, rather than concentrate on rigorous proofs of theorems. Most of the important results are followed by examples that illustrate the uses and implications of the results. Many of these examples are drawn from the physical sciences, including detailed treatments of potential theory, steady state temperature problems, hydrodynamics, seepage flows, electrostatics, gravitation and the use of the Laplace transform for solving the heat conduction and wave propagation equations. I do include some proofs that reflect the fundamental spirit of complex function theory, such as the necessary and sufficient conditions for the existence of complex derivatives, the Cauchy–Goursat integral theorem, Cauchy integral formulas, Taylor series theorem and Laurent series theorem. However, my main concern is to prepare students with the fundamental concepts of complex variable theory and the manipulative skills to apply complex variable methods to physical problems.

Throughout the whole text, both algebraic and geometric tools are employed to provide the greatest understanding, with many diagrams illustrating the concepts introduced. The text contains some 300 stimulating exercises, with solutions given to many of them. They are intended to aid students to grasp the concepts covered in the text and foster the skills in applying complex variables techniques to solve physical problems. Students are strongly advised to work through as many exercises as possible since mathematical knowledge can only be gained through active participation in the thinking and learning process.

The book begins by carefully exploring the algebraic, geometric and topological structures of the complex number field. In order to visualize the complex infinity, the Riemann sphere and the corresponding stereographic projection are introduced.

Analytic functions are introduced in Chapter 2. The highlights of the chapter are the Cauchy–Riemann relations and harmonicity. The use of complex functions in describing fluid flows and heat distributions are illustrated.

In Chapter 3, the complex exponential function is introduced as an entire function which is equal to its derivative. The description of steady state temperature distributions by complex logarithm functions is illustrated. The mapping properties of complex trigonometric functions are examined. The notion of Riemann sheets is introduced to help visualize multi-valued complex functions.

Complex integration forms the cornerstone of complex variable theory. The key results in Chapter 4 are the Cauchy–Goursat theorem and the Cauchy integral formulas. Other interesting results include Gauss' mean value theorem, Liouville's theorem and the maximum modulus theorem. The link of analytic functions and complex integration with the study of conservative fields is considered. Complex variable methods are seen to be effective analytical tools to solve conservation field models in potential flows, gravitational potentials and electrostatics.

Complex power series are the main themes in Chapter 5. The Taylor series theorem and Laurent series theorem show that a convergent power series is an analytic function within its disc of convergence. The notion of analytic continuation is discussed. As an application, the solution to the potential flow over a perturbed circle is obtained as a power series in a perturbation parameter.

In Chapter 6, we start with the discussion of the classification of isolated singularities by examining the Laurent series expansion in a deleted neighborhood of the singularity. We then examine the theory of residues and illustrate the applications of the calculus of residues in the evaluation of complex integrals. The concept of the Cauchy principal value of an improper integral is introduced. Fourier transforms and Fourier integrals are considered. The residue calculus method is applied to compute the hydrodynamic lift and moment of an immersed obstacle.

The solutions of boundary value problems and initial-boundary value problems are considered in Chapter 7. The Poisson integral formula and the Schwarz integral formula for Dirichlet problems are derived. The inversion of Laplace transform via the Bromwich contour integral is discussed. Laplace transform techniques are applied to obtain the solutions of initial-boundary value problems arising from heat conduction and wave propagation models.

In the last chapter, we explore the rich geometric structure of complex variable theory. The geometric properties associated with mappings represented by complex functions are examined. The link between analyticity and conformality is derived. Various types of transformations that perform the mappings of regions are introduced. Bilinear and Schwarz–Christoffel transformations are discussed fully, and it is shown how to use them to transform conservative field problems with complicated configurations into those with simple geometries.

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